

MODEL ANSWER

1) *Determine the point group.*

BF₃ is in the D_{3h} point group.

2) *Degrees of freedom.*

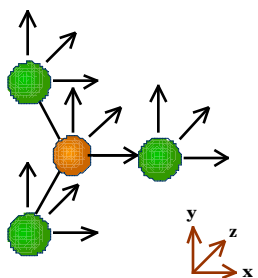
BF₃ is a non-linear molecule, with 4 atoms.

Using the equation $3N$, we see that BF₃ has 12 degrees of freedom.

Using the equation $3N - 6$, we see that BF₃ has $(12 - 6 =) 6$ vibrational degrees of freedom.

3) *Determine irreducible representations of Γ_{tot} .*

Three axes put on each atom. $\Gamma_{4\text{ atoms}}$ calculated by seeing the effect on the axes by all the symmetry operations.



	E	2C ₃	3C ₂	σ _h	2S ₃	3σ _v
$\Gamma_{4\text{ atoms}}$	12	0	-2	4	-2	2

The contributions from each symmetry species are as follows.

$$A_1': \quad 1/12 [(12 \times 1 \times 1) + (0 \times 1 \times 2) + (-2 \times 1 \times 3) + (4 \times 1 \times 1) + (-2 \times 1 \times 2) + (2 \times 1 \times 3)]$$

$$= \quad 1/12 [12 - 6 + 4 - 4 + 6] \quad = \quad 1$$

$$A_2': \quad 1/12 [(12 \times 1 \times 1) + (0 \times 1 \times 2) + (-2 \times -1 \times 3) + (4 \times 1 \times 1) + (-2 \times 1 \times 2) + (2 \times -1 \times 3)]$$

$$= \quad 1/12 [12 + 6 + 4 - 4 - 6] \quad = \quad 1$$

$$E': \quad 1/12 [(12x_2x_1) + (0x_1x_2) + (-2x_0x_3) + (4x_2x_1) + (-2x_1x_2) + (2x_0x_3)]$$

$$= \quad 1/12 [24 + 8 + 4] \quad = \quad 3$$

$$A_1'': \quad 1/12 [(12x_1x_1) + (0x_1x_2) + (-2x_1x_3) + (4x_1x_1) + (-2x_1x_2) + (2x_1x_3)]$$

$$= \quad 1/12 [12 - 6 - 4 + 4 - 6] \quad = \quad 0$$

$$A_2'': \quad 1/12 [(12x_1x_1) + (0x_1x_2) + (-2x_1x_3) + (4x_1x_1) + (-2x_1x_2) + (2x_1x_3)]$$

$$= \quad 1/12 [12 + 6 - 4 + 4 + 6] \quad = \quad 2$$

$$E'': \quad 1/12 [(12x_2x_1) + (0x_1x_2) + (-2x_0x_3) + (4x_2x_1) + (-2x_1x_2) + (2x_0x_3)]$$

$$= \quad 1/12 [24 - 8 - 4] \quad = \quad 1$$

$$\text{Therefore } \Gamma_{tot} = A_1' + A_2' + 3E' + 2A_2'' + E''$$

Γ_{tot} has twelve degrees of freedom. This agrees with our earlier answer.

3) Determine Γ_{vib} .

$$\text{We know that } \Gamma_{tot} = \Gamma_{trans} + \Gamma_{rot} + \Gamma_{vib}$$

From character table we see that...

$$\Gamma_{trans} = E' + A_2''$$

$$\Gamma_{rot} = A_2' + E''$$

$$\text{Therefore using } \Gamma_{vib} = \Gamma_{tot} - \Gamma_{trans} - \Gamma_{rot}$$

$$\Gamma_{vib} = A_1' + 2E' + A_2''$$

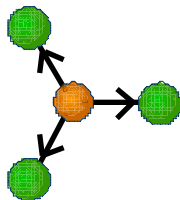
Γ_{vib} has six degrees of freedom. This agrees with our earlier answer.

4) Split into stretches and bends.

BF_3 has three bonds, so therefore has 3 stretches and 3 bends.

5) *Determine irreducible representations of $\Gamma_{stretch}$.*

One axis put on each bond. Bond calculated by seeing the effect on the axes by all the symmetry operations.



	E	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_v$
$\Gamma_{3 \text{ bonds}}$	3	0	1	3	0	1

The contributions from each symmetry species are as follows.

$$A_1': \quad 1/12 [(3 \times 1 \times 1) + (0 \times 1 \times 2) + (1 \times 1 \times 3) + (3 \times 1 \times 1) + (0 \times 1 \times 2) + (1 \times 1 \times 3)]$$

$$= \quad 1/12 [3 + 3 + 3 + 3] \quad = \quad 1$$

$$A_2': \quad 1/12 [(3 \times 1 \times 1) + (0 \times 1 \times 2) + (1 \times -1 \times 3) + (3 \times 1 \times 1) + (0 \times 1 \times 2) + (1 \times -1 \times 3)]$$

$$= \quad 1/12 [3 - 3 + 3 - 3] \quad = \quad 0$$

$$E': \quad 1/12 [(3 \times 2 \times 1) + (0 \times -1 \times 2) + (1 \times 0 \times 3) + (3 \times 2 \times 1) + (0 \times -1 \times 2) + (1 \times 0 \times 3)]$$

$$= \quad 1/12 [6 + 6] \quad = \quad 1$$

$$A_1'': \quad 1/12 [(3 \times 1 \times 1) + (0 \times 1 \times 2) + (1 \times 1 \times 3) + (3 \times -1 \times 1) + (0 \times -1 \times 2) + (1 \times -1 \times 3)]$$

$$= \quad 1/12 [3 + 3 - 3 - 3] \quad = \quad 0$$

$$A_2'': \quad 1/12 [(3 \times 1 \times 1) + (0 \times 1 \times 2) + (1 \times -1 \times 3) + (3 \times -1 \times 1) + (0 \times -1 \times 2) + (1 \times 1 \times 3)]$$

$$= \quad 1/12 [3 - 3 - 3 + 3] \quad = \quad 0$$

$$E'': \quad 1/12 [(3 \times 2 \times 1) + (0 \times -1 \times 2) + (1 \times 0 \times 3) + (3 \times -2 \times 1) + (0 \times 1 \times 2) + (1 \times 0 \times 3)]$$

$$= \quad 1/12 [6 - 6] \quad = \quad 0$$

Therefore $\Gamma_{stretch} = A_1' + E'$

6) *Determine Γ_{bend} .*

We know that $\Gamma_{bend} = \Gamma_{vib} - \Gamma_{stretch}$

Therefore $\Gamma_{bend} = E' + A_2''$

7) *Assign irreducible representations to spectra.*

From character tables we see that only E' will be visible in both IR and Raman spectra.

E' stretch will be at higher energy (1505 cm^{-1})

E' bend will be at lower energy (482 cm^{-1})

From character tables we see that A_1' will not be visible in the IR.

A_1' stretch will be at 888 cm^{-1}

From character tables we see that A_2'' will not be visible in Raman.

A_2'' bend will be at 718 cm^{-1}

1505 (R, IR)

E' stretches

888 (R)

A_1' stretch

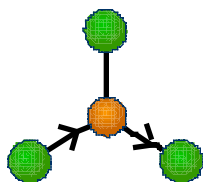
718 (IR)

A_2'' bend

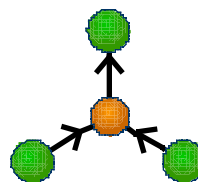
482 (R, IR)

E' bends

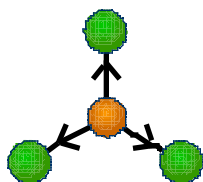
8) *Sketch these vibrations (although not specifically asked for, this is a common question).*



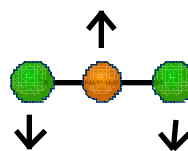
E' stretch



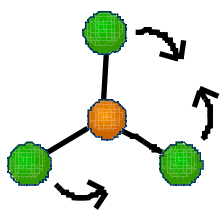
E' stretch



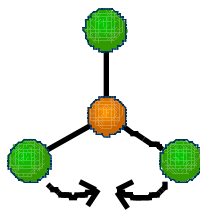
A_1' stretch



A_2'' bend



E' bend



E' bend